Check-Up

12

The distributive laws

There are A pupils in a class. The school charges them each £B to cover the cost of transport for a field trip, plus an additional £C to cover other costs.

a) Without a calculator, find the total amount collected if A = 30, B = 25 and C = 6.

b) Which of these formulas gives the total amount collected in pounds: A(B + C) or AB + AC?

Answers to check-up 12

a) £930. b) Either formula can be used.

Discussion and explanation to check-up 12

Multiplication is said to be *distributive* over addition or subtraction. What this means is that if you have to multiply the sum (or difference) of two numbers B and C by a third number A, then you can multiply them separately by A and then find the sum (or difference) of the results.

See how this works with the example here. To work out the total amount collected you could first work out the sum of B and C, i.e. the amount paid by each pupil (£25 + £6 = £31). Then multiply this by A, the number of pupils (30 £31 = £930). The formula used here is A(B + C). Alternatively, you could work out separately how much is collected for transport (30 £25 = £750) and how much is collected for other costs (30 £6 = £180). Then add these: £750 + £180 = £930. The formula used here is AB + AC. These procedures come to the same result. They always do, whatever numbers are used. Written algebraically, the distributive laws for multiplication state that for any numbers A, B and C: A(B + C) = AB + AC and A(B – C) = AB – AC. Remember that AB is shorthand for 'A multiplied by B', and A(B + C) means 'A multiplied by the sum of B and C'.

These distributive laws are the basis of many approaches to multiplication calculations, both written and mental. For example, if you had to work out the cost of 28 textbooks at £9 each, you might handle the multiplication mentally by splitting the 28 into 20 + 8, like this: 9 28 = 9 (20 + 8) = (9 20) + (9 8) = 180 + 72 = 252. The multiplication by 9 has been 'distributed' across the sum of 20 and 8. Or, you could rewrite the 28 mentally as 30 – 2, like this: 9 28 = 9 (30 – 2) = (9 30) – (9 2) = 270 – 18 = 252. This time the multiplication by 9 has been distributed across the difference of 30 and 2.

Division by a number can also be distributed across a sum or difference, giving us these two distributive laws: (B + C) ÷ A = (B ÷ A) + (C ÷ A) and (B – C) ÷ A = (B ÷ A) – (C ÷ A). (Of course, these don't make sense if A = 0.) For example, 171 ÷ 9 could be handled by distributing the division by 9 across the sum of 99 and 72, like this: 171 ÷ 9 = (99 + 72) ÷ 9 = (99 ÷ 9) + (72 ÷ 9) = 11 + 8 = 19. Or, the division by 9 could be distributed across the difference of 180 and 9, like this: 171 ÷ 9 = (180 – 9) ÷ 9 = (180 ÷ 9) – (9 ÷ 9) = 20 – 1 = 19. In both cases, I have chosen to rewrite the 171 in terms of numbers that I can easily divide by 9 (i.e. 99 and 72, 180 and 9).

Summary of key ideas

- Multiplication is said to be *distributive* over addition or subtraction.
- This means that to multiply the sum (or difference) of two numbers by something you can multiply them separately and then find the sum (or difference) of the results.
- Algebraically, for any numbers A, B and C:
  $A(B + C) = AB + AC$, and $A(B – C) = AB – AC$.
- Division by a number is also distributive across addition and subtraction.
- Algebraically, for any numbers A, B and C:
  $(B + C) \div A = (B \div A) + (C \div A)$ and $(B – C) \div A = (B \div A) – (C \div A)$
  [provided A does not equal 0].
- These laws are used extensively in multiplication and division calculations.

Further practice

12.1 Use the distributive laws to calculate mentally the cost of 180 textbooks at £8 each, by thinking of the 180 as (a) 100 + 80, (b) 200 – 20.

12.2 Find mentally the total cost of equipping 25 pupils with a textbook costing £12 and a workbook costing £4:

a) using the process represented by A(B +C)

b) using the process represented by AB + AC.

Describe in words the two processes.

12.3 The headteacher of a primary school receives additional funding of £1330 from the PTA for reading books, to be distributed equally across seven year groups. How much is this per year group? Work this out mentally, using the distributive laws of division, by thinking of the £1330 as (a) £700 add something, (b) £1400 subtract something.